AVIRAL CLASSES
IIT-JEE | NEET | FOUNDATIONS

ULTIMATE TEST SERIES JEE MAIN -2020
TEST-03 ANSWER KEY
Test Date :05-03-2020

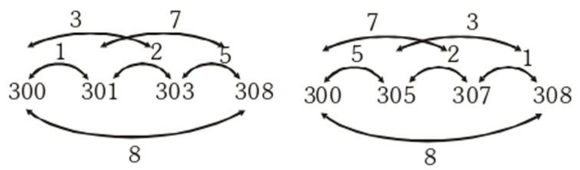
[PHYSICS]

1. $PV = nRT \;\&\; PV^2 = \text{constant} \Rightarrow V \propto \frac{1}{T}$

$\Rightarrow$ gas can expand only if it cools .

As tempeature decreases during expansion so internal energy will decrease.

2.



3. $W_{net} = W_{1\text{-}2} + W_{2\text{-}3} + W_{3\text{-}4} + W_{4\text{-}1}$

$= 0 + \mu R\,(T_3 - T_2) + 0 + \mu R(T_1 - T_4)$

$= \mu R[T_3 - T_2 + T_1 - T_4] = 20$ kJ

4. $V = \frac{\omega}{K} = \frac{\frac{\pi}{2}\times 8}{\frac{\pi}{2}\times\frac{1}{8}} = 64$

5. B

6. $A = 2$ and $KA = 8 \quad \Rightarrow K = \frac{8}{2} = 4$

$T = 2\pi\sqrt{\frac{M}{K}} = 2\pi\sqrt{\frac{0.01}{4}}$

7. $n_1 = 256 \times \frac{V}{V - \frac{V}{20}} = 256 \times \frac{20}{19} = 269.5$ Hz

$n_2 = 256 \times \frac{V}{V + \frac{V}{20}} = 256 \times \frac{20}{21} = 243.8$ Hz

8. The force is determined from the relation

$F = -\frac{dU}{dx} = -\frac{d}{dx}(2.5x^2 + 100)$

$F = -5x$ newton

The motion is simple harmonic because $F \propto -x$

compare the above relation with $F = -Kx$

force constant $K = 5\,\frac{N}{m}$

Therefore time period

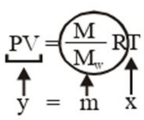
$T = 2\pi\sqrt{\frac{0.2}{5}} = \frac{2\pi}{5} \approx 1.26$sec.

9. $\frac{n_1}{n} = \sqrt{\frac{T_1}{T}} = \sqrt{\frac{15}{16}}$

$n_1 = 320\sqrt{\frac{15}{16}} \approx 310$ Hz

10. C

11.



slope $\propto \frac{1}{M_w}$ (M = const.)

12. $dU_I = dU_{II}$

$\Delta Q_I - \Delta W_I = \Delta Q_{II} - \Delta W_{II}$

$8 \times 10^5 - 6.5 \times 10^5 = 10^5 - \Delta W_{II}$

$\Delta W_{II} = -0.5 \times 10^5$ J

work done on the gas $= 0.5 \times 10^5$ J

13. $v = \omega\sqrt{a^2 - x^2} \Rightarrow x = \sqrt{a^2 - \frac{v^2}{\omega^2}}\ (\because \omega^2 = \frac{k}{m})$

$$x = \sqrt{a^2 - \frac{v^2 m}{k}} = \sqrt{(0.5)^2 - \frac{(0.4)^2 \times (10)}{(10)}}$$

$$= \sqrt{0.25 - 0.16} = \sqrt{0.09} = 0.3\text{m}$$

14. $\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$

at $x = 0$, K.E. $= \frac{1}{2}mv^2_{max} = \frac{1}{2}m\omega^2 A^2$

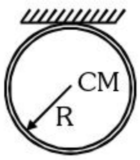
at $x = \frac{A}{2}$, K.E. $= \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2\left[A^2 - \left(\frac{A}{2}\right)^2\right]$

$$= \frac{3}{4}\left[\frac{1}{2}m\omega^2 A^2\right] \Rightarrow \frac{KE_{x=0}}{KE_{x=A/2}} = \frac{4}{3}$$

15. $I \propto A^2$

$$\therefore \frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \frac{16}{4} = 4:1$$

16.



$$T = 2\pi\sqrt{\frac{\ell + \frac{K^2}{\ell}}{g}}$$

Here $\ell = R$, $MK^2 = MR^2 \Rightarrow K = R$

$$\Rightarrow T = 2\pi\sqrt{\frac{R+R}{g}} = 2\pi\sqrt{\frac{2R}{g}}$$

17. $PV = \frac{M}{M_W}RT$

$P \propto MT$

$$\Rightarrow \frac{P_1}{P_2} = \frac{M_1 T_1}{M_2 T_2}$$

$$\Rightarrow \frac{720}{P_2} = \frac{M}{3M/4} \times \frac{313}{626}$$

$P_2 = 1080$ kp_a

18. $\eta = \frac{W}{Q} = 1 - \frac{T_2}{T_1}$

$$= \frac{W}{6} = 1 - \frac{400}{500}$$

$\Rightarrow W = 1.2$ Kcal

19. $\Delta Q = \Delta W + dU$

$100 = 20 + dU$

$dU = 80$ J

For Reverse

$dU_r = -80$

$\Rightarrow \Delta Q_r = \Delta W_r + dU_r$

$-20 = \Delta W_r - 80$

$\Delta W_r = 60$ J

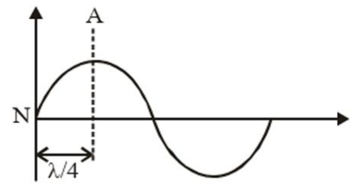
20. $\frac{Q}{t} = e_r\sigma A(T_4 - T_0^4)t$

$= 0.4 \times 5.67 \times 10^{-8} \times 200 \times 10^{-4}\ [800^4 - 300^4]$

$= 182$ J/sec.

INTEGER

21.



$$n = \frac{1}{2\ell}\sqrt{\frac{T}{\pi r^2 d}}$$

22. D

23. $\frac{E_T}{V} = \frac{3}{2} P = \frac{3}{2} \times 2 \times 10^5 = 3 \times 10^5$ J

24. $n_1 \sim n_2 = 4$

$T_b = \frac{1}{n_1 - n_2} = \frac{1}{4}$ sec.

25. $\eta = \frac{W}{Q_1} = \frac{T_1 - T_2}{T_1}$

$\Rightarrow \frac{W}{1000} = \frac{684 - 342}{684}$

$\Rightarrow W$

[CHEMISTRY]

26. Acetanilide act as negative catalyst for decomposition of H_2O_2

($H_2O_2 \xrightarrow{\text{Ace tan ilide}} H_2O + [O]$)

27. B
28. D
29. B
30. C
31. A
32. D
33. D
34. D
35. B
36. D
37. B
38. A
39. ALUM : $M'_2SO_4.M''_2(SO_4)_3.24H_2O$ (x = 24)
40. B
41. D

42. $XeF_4 \xrightarrow{H_2O} XeO_3$ on complete hydrolysis

43. A
44. D
45. A

INTEGER

46. 1
47. 4
48. 1
49. 1
50. 6

[MATHEMATICS]

51 **Ans. (2)**

$PA \times PB = (PT)^2$

where PT = length of tangent

$(PT)^2 = (-1)^2 + 3^2 - 2(-1) + 4(3) - 8 = 16$

$P(A)\,P(B) = 16$

$\therefore AM \geq GM$

$PA + PB \geq 8$

52 **Ans. (4)**

$ax + by + c = 0$

$a + c = 2b \Rightarrow a - 2b + c = 0$

$x = 1, y = -2$

$(1, -2) = (\alpha, \beta)$

$(x - 1)^2 + (y + 2)^2 = \gamma$

$\Rightarrow x^2+y^2-2x + 4y + 5 - \gamma = 0$

it is orthogonal to $x^2+y^2 - 4x - 4y - 1 = 0$

$\Rightarrow 4 - 8 = 5 - \gamma - 1$

$\gamma = 8$

$\alpha + \beta + \gamma = 1 - 2 + 8 = 7$

53 **Ans. (1)**

The normal at the extremities of focal chord meet at right angle. So orthocentre is the point of intersection of normals.

If $P(at_1^2, 2at_1), Q(at_2^2, 2at_2)$ then $t_1t_2 = -1$,.

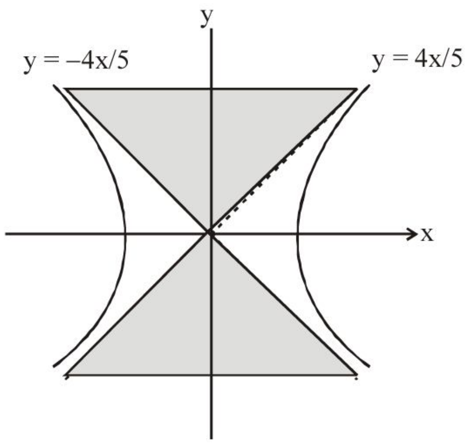
Point of intersection of normals

$h = a(t_1^2 + t_2^2 + t_1t_2 + 2)$

$k = -at_1t_2(t_1 + t_2)$

54 **Ans. (2)**

Region where 2 tangents to two different branches can be drawn.



$\therefore (1, 6), (1, 3)$

But from (1, 6) 2 tangents to circle can be drawn

$\therefore$ Ans. (1, 3)

55 **Ans. (2)**

ends of L.R $\left(\pm ae, \pm \frac{b^2}{a}\right)$

$\Rightarrow$ tangents are $\pm \frac{e}{a}x \pm \frac{1}{a}y = 1$

$\Rightarrow$ Area $= \frac{2a^2}{e} - a^2e^2$

56. **Ans. (2)**

$$\lim_{x^2\to a}\frac{b-\cos\left(x^2-a\right)}{\left(x^2-a\right)\sin\left(x^2-a\right)}$$

Let $x^2 - a = t$

$$\lim_{t\to 0}\frac{b-\cos t}{t\sin(c(t+a)-a)}$$

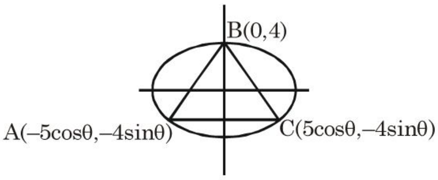
$$\Rightarrow \frac{b-1}{0} \quad \Rightarrow \quad b=1$$

$$\lim_{t\to 0}\frac{1-\cos t}{t\sin(ct+a(c-1)} = \lim_{t\to 0}\frac{2\sin^2\frac{t}{2}}{t\sin(ct+a(c-1))}$$

$$\lim_{t\to 0}\frac{\sin\frac{t}{2}}{\sin\left(ct+a(c-1)\right)} = \frac{0}{\sin a(c-1)} \Rightarrow \quad c=1$$

$$\lim_{t\to 0}\frac{\sin\frac{t}{2}}{\sin t} = \frac{1}{2} \quad \Rightarrow \quad L=\frac{1}{2}$$

57. **Ans. (3)**

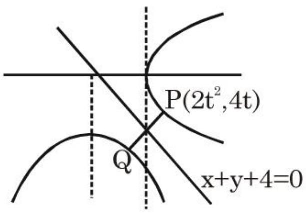


$$\text{Area } = \frac{1}{2}10\cos\theta\left(4+4\sin\theta\right)$$

$$\frac{dA}{d\theta} = 0 \Rightarrow \theta = \frac{\pi}{6}$$

$$A_{max} = 15\sqrt{3}$$

58. **Ans. (1)**



for minimum distance $\left.\frac{dy}{dx}\right|_P = -1$

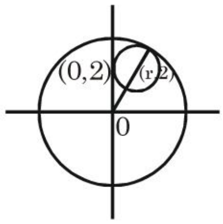
$\Rightarrow t = -1$

$\Rightarrow$ min distance = PQ = $2\sqrt{2}$

59. **Ans. (2)**

$$4 = \sqrt{r^2+4} + r$$

$$\Rightarrow r = \frac{3}{2}$$



60. **Ans.(2)**

Given conic C is parabola

focus : (1,–1)

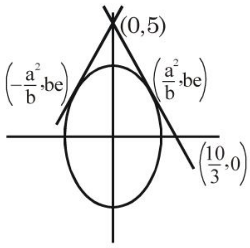
E : 3x – 4y = 0

minimum distance is ⊥ distance

$$\Rightarrow \text{distance } = \left|\frac{6+4}{5}\right| = 2$$

61. **Ans. (4)**

Let ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$



tangent $\frac{x}{b}+\frac{ye}{b}=1$

$$\therefore \frac{b}{e}=5$$

$$\text{and } b=\frac{10}{3} \Rightarrow e=\frac{2}{3}$$

$$\Rightarrow a^2 = \frac{500}{81} \quad \therefore \text{ L.R} = \frac{2a^2}{b} = \frac{100}{27}$$

62. **Ans. (2)**

If circle intersect at 4 points then

sum of x coor = 0

$\therefore$ Points (17,289), (–2,4), (13,169),(–28,784)

directrix is $y = -\frac{1}{4}$

sum of perpendicular distances

$$= \left(289+\frac{1}{4}\right)+\left(4+\frac{1}{4}\right)+\left(169+\frac{1}{4}\right)+\left(784+\frac{1}{4}\right)$$

$= 1247$

63. **Ans. (3)**

Differentiate both sides wrt 'x',

$$(e-1)e^{xy}\left(\frac{xdy}{dx}+y\right)+2x = e^{x^2+y^2}\left(2x+2y\frac{dy}{dx}\right)$$

$$(e-1)\left(\frac{dy}{dx}\right)\Bigg|_{(1,0)}+2 = e(2)$$

$$\frac{dy}{dx}\Bigg|_{(1,0)} = 2$$

64. **Ans. (4)**

$\alpha = 135^\circ$

$$\lim_{\theta\to\frac{3\pi^+}{4}} \frac{\tan\theta - 1}{[\sin\theta + \cos\theta]} = \frac{-2}{-1} = 2$$

65. **Ans. (3)**

$h(x) = f(g(f(x))$

$h'(x) = f'(g(f(x)).g'(f(x)).f'(x)$

$h'(2) = 64.$

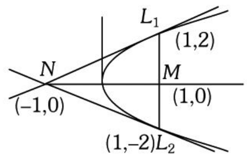
66. B

67. -

68. A

69. Equation to the tangent at (x_1, y_1) on the parabola $y^2 = 4ax$ is $yy_1 = 2a(x + x_1)$. In this case, $a = 1$

The co-ordinates at the ends of the latus rectum of the parabola $y^2 = 4x$ are $L(1,2)$ and $L_1(1,-2)$.



Equation of tangents at L and L_1 are $2y = 2(x+1)$ and $-2y = 2(x+1)$, which gives $x = -1,\ y = 0$.

Thus the required point of intersection is $(-1, 0)$.

70. Equation of focal chord for the parabola $y^2 = 4ax$, passes through the point $(au^2, 2au)$ and $(av^2, 2av)$

$$\Rightarrow y - 2au = \frac{2av - 2au}{av^2 - au^2}(x - au^2)$$

$$\Rightarrow y - 2au = \frac{2a(v-u)}{a(v-u)(v+u)}(x - au^2)$$

$$\Rightarrow y - 2au = \frac{2}{(v+u)}(x - au^2)$$

If this is focal chord, so it would passes through focus $(a,0)$.

$\Rightarrow 0 - 2au = \frac{2}{v+u}(a - au^2) \Rightarrow -uv - u^2 = 1 - u^2$,

$\therefore uv + 1 = 0.$

INTEGER

71. **Ans. (4)**

$$\lim_{x\to\frac{1}{2}} \frac{ax^2+bx+c}{(2x-1)^2} = \frac{1}{2}$$

$$\Rightarrow ax^2+bx+c = \frac{1}{2}(2x-1)^2$$

$$\Rightarrow ax^2+bx+c = 2x^2-2x+\frac{1}{2}$$

$$\therefore a = 2, b = -2, c = \frac{1}{2}$$

$$\therefore \lim_{x\to 2} \frac{(x-2)(x+2)\left(x-\frac{1}{2}\right)}{x-2} = 4\times\frac{3}{2} = 6$$

72. **Ans. (4)**

$$\frac{2x^3+3x^2+x-3}{x^2+x-2}=(2x+1)+\frac{1}{x-1}+\frac{3}{x+2}$$

$$\frac{d}{dx}\left[\frac{2x^3+3x^2+x-3}{x^2-x-2}\right]=2-\frac{1}{(x-1)^2}-\frac{3}{(x+2)^2}$$

$\therefore \mathrm{A}=2, \mathrm{B}=-1, \mathrm{C}=-3$

$\therefore \mathrm{A}+\mathrm{B}+\mathrm{C}=0$

73. Solving $3x+4y=9, y=mx+1$ we get $x=\dfrac{5}{3+4m}$

x is an integer if $3+4m=1,-1,5,-5$

$\therefore\ m=\dfrac{-2}{4},\dfrac{-4}{4},\dfrac{2}{4},\dfrac{-8}{4}$. So, m has two integral values.

74. The given lines are $\pm x \pm y = 1$

i.e. $x+y=1, x-y=1, x+y=-1$ and $x-y=-1$

These lines form a quadrilateral whose vertices are $A(-1,0), B(0,-1), C(1,0)$ and $D(0,1)$

Obviously $ABCD$ is a square.

Length of each side of this square is $\sqrt{1^2+1^2}=\sqrt{2}$

Hence area of square is $\sqrt{2}\times\sqrt{2}=2 sq. units$

The co-ordinates of foci are $(\pm ae, 0)$. Here $a=4,\ b=3$

$\therefore\ b^2=a^2(1-e^2) \Rightarrow 9=16(1-e^2) \Rightarrow \dfrac{9}{16}=1-e^2$

75. $\Rightarrow e=\pm\sqrt{\dfrac{7}{4}}$; $\therefore$Points are $(\pm\sqrt{7},0)$.

$\therefore$ Radius $=\sqrt{(\sqrt{7}-0)^2+(0-3)^2}=\sqrt{7+9}=\sqrt{16}=4$.